Inflation - Part 1

Overview

- Definitions, symbols
- Trends in recent history
- Examples of price indices and conversions
- Implications for MARR

Inflation - Part 1

Some definitions

- Inflation - an increase in the price of goods and services with no change in quality
- Price index - a measure of this type of inflationary increase with respect to a “baseline"
- Constant dollars - a concept for evaluating cash flows in terms of purchasing power based on a “baseline” year
- Actual dollars - recorded amounts

Inflation - Part 1

Some examples

- Inflation - a ten-speed bicycle that cost $90 in 1970 costs $218 in 2000. This represents an annual increase of \( f = 3\%: $90(1 + 0.03)^{30} = \$218 \)

Inflation - Part 1

Some more examples

- Constant dollars - The price of a ten-speed bicycle in 2000 is $90 in year-1970 constant dollars.
- Constant dollars - The cost of a hospital stay of five days in 1998, that costs $3,000 in actual dollars, is $3,000/2.381 = $1,260 in year 1983 constant dollars.

Inflation - Part 1

Another example

The price index values for lumber for 1989 and 1999 are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Index Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989</td>
<td>125.7</td>
</tr>
<tr>
<td>1999</td>
<td>179.5</td>
</tr>
</tbody>
</table>

A typical 3,000 sq.ft. home required $15,000 in lumber in 1989. In 1999 such a home would require:

\[
$15,000(179.5/125.7) = 15,000(1.428) = \$21,420
\]
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Examples of indices

<table>
<thead>
<tr>
<th>Time after 1987</th>
<th>Foods</th>
<th>Motor vehicles</th>
<th>All consumer</th>
<th>Medical care</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
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</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observation 1

- The earnings rate on U.S. Treasury Bills is approximately 2.5 - 3.0% above the general inflation rate.
- The earnings rate on common stock is approximately 6 - 7% above the general inflation rate.

More definitions

- Market interest rate - the interest rate earned on actual dollars in the financial market, \( i \)
  - Also called: nominal rate, inflation-adjusted rate
- “Real” interest rate - a measure of the earning power of money measured in constant dollars, \( i' \)
  - Also called: inflation-free rate

Fundamental relationship

The market interest rate factor is equal to the “real” interest rate factor multiplied by the inflation rate factor:

\[
(1 + i')(1 + f) = (1 + i)
\]

where

- \( i \) = market, or actual rate
- \( f \) = inflation rate
- \( i' \) = “real” interest rate

Example of interest rates

Market rate \( i = 12\% \)
Inflation rate \( f = 3.5\% \)

“Real” interest rate

\[
= \frac{(1 + 0.12)}{(1 + 0.035)} - 1
= 1.08213 - 1 = 8.213\% 
\]

This reflects the increase in purchasing power each year.

Example of MARR

MARR = 20% when \( f = 3\% \)

“Real” MARR

\[
= \frac{(1 + 0.20)}{(1 + 0.03)} - 1
= 1.165 - 1 = 16.5\%
\]

If inflation forecast is \( f = 6\% \)

MARR, for actual dollar analysis should be

\[
(1 + 0.165)(1 + 0.06) - 1
= 0.23495 = 23.5\%
\]
Inflation - Part 1

Example 1

We wish to deposit a sum of money now into an account that earns 8% to provide for retirement home care for a parent for the next eight years. The annual expenses currently are $22,000. The price index for this care is forecasted to increase 5% per year. How much must we deposit now into the account?

The actual $ expenses as forecasted:
year 1: \(22,000(1+0.05)^1 = 23,100\)
year 2: \(22,000(1+0.05)^2 = 24,255\)
year 3: \(22,000(1+0.05)^3 = 25,468\)
year 7: \(22,000(1+0.05)^7 = 30,956\)
year 8: \(22,000(1+0.05)^8 = 32,504\)

We need to deposit now the amount
\[ P = 23,100(1+0.08)^{-1} + 24,255(1+0.08)^{-2} + 25,468(1+0.08)^{-3} + \ldots + 30,956(1+0.08)^{-7} + 32,504(1+0.08)^{-8} = \$155,369 \]

Example 1, constant $

The constant $ expenses as forecasted:
year 1: 22,000
year 2: 22,000
year 8: 22,000

Here the “real” interest rate can be
\[ i' = \frac{(1.08)}{(1.05)} - 1 = 0.0286 \]

We need to deposit now the amount
\[ P = 22,000(1+0.0286)^{-1} + 22,000(1+0.0286)^{-2} + \ldots + 22,000(1+0.0286)^{-8} = \$155,369 \]

Observation 1

Because “real” interest rates tend to remain fairly stable, while inflation rates vary, it is often easier to perform analysis using constant dollars.

Any Present Worth amount computed this way would tend to be the same, regardless of the inflation rate.

Holds for stable economies, ignores effect on depreciation.

Observation 2

Both methods of analysis, actual dollar analysis and constant dollar analysis, lead to the same numerical Present Worth amount.

The key to this result is consistency between the market interest rate and the “real” interest rate, related via the inflation rate.

Observation 3

Because “real” interest rates tend to remain fairly stable, while inflation rates vary, it is often easier to perform analysis using constant dollars.

Any Present Worth amount computed this way would tend to be the same, regardless of the inflation rate.

Holds for stable economies, ignores effect on depreciation.

Definition

- inflation, price indices
- “real” interest rate
- constant $, actual $

Examples

- with price indices
- PW: constant $, actual $

Observations

- stability of “real” rates
- convenience of PW using constant $
Inflation - Part 2

Example 2

We wish to make annual deposits into a savings account so that the accumulated value is $1,000,000 for retirement purposes.

Time horizon: 30 years
“Real” interest rate: 4%
Inflation rate: 3%

The value of $1,000,000 is in constant dollars.
How much is the annual deposit?

Example 2 - method 1

Perform analysis in constant dollars and apply the \( A/F, i, N \) factor to obtain a uniform series of deposits measured in constant dollars.

\[
A = 1,000,000 \cdot \left( \frac{A}{F}, 4\%, 30 \right)
\]
\[
= 1,000,000 \cdot 0.017830
\]
\[
= $17,830
\]

Correction: \( A/F \)

The actual $ deposits will be:
year 1: \((17,830)(1+0.03)\)\(^1\) = 18,365
year 2: \((17,830)(1+0.03)\)\(^2\) = 18,916
year 3: \((17,830)(1+0.03)\)\(^3\) = 19,483

\downarrow
year 29: \((17,830)(1+0.03)\)\(^{29}\) = 42,018
year 30: \((17,830)(1+0.03)\)\(^{30}\) = 43,278

The actual $ accumulation will be:
year 1: \((18,365)(1+0.0712)\)\(^1\) = 19,071
year 2: \((18,916)(1+0.0712)\)\(^2\) = 20,642
year 3: \((19,483)(1+0.0712)\)\(^3\) = 22,245

\downarrow
year 29: \((42,018)(1+0.0712)\)\(^{29}\) = 75,018
year 30: \((43,278)(1+0.0712)\)\(^{30}\) = 81,743

Sum = $2,427,262

Check:
$1,000,000(1 + 0.03)^{30} = $2,427,262 ✓
Perform analysis in actual dollars and apply the \((A/F, i, N)\) factor to obtain a uniform series of deposits measured in actual dollars. 
(This series will be different from the other one, no matter how it is measured, in constant $ or actual $.)

First obtain desired accumulation:
\[
F = \$1,000,000(1 + 0.03)^{30} = \$2,427,262
\]

The expected market rate is 
\[
i = (1 + 0.04)(1 + 0.03) - 1 = 0.0712 = 7.12%
\]

The annual deposit then is 
\[
A = \frac{\$2,427,262(A/F, 7.12\%, 30)}{2,427,262(0.010360)} = \$25,146
\]

The actual $ accumulation will be:
- Year 1: \((25,146)(1+0.0712)^{29} = 184,809\)
- Year 2: \((25,146)(1+0.0712)^{28} = 172,525\)
- Year 3: \((25,146)(1+0.0712)^{27} = 161,058\)
- Year 29: \((25,146)(1+0.0712)^{1} = 26,937\)
- Year 30: \((25,146)(1+0.0712)^{0} = 25,146\)

\[\text{Sum} = \$2,427,262\]

Note: The purchasing power of the deposit declines every year.

The two approaches result in completely different deposit series.
Other series are possible, such as arithmetic gradient, geometric gradient in terms constant dollars, etc.
If a complicated analysis is likely, spreadsheets are recommended.
Inflation - Part 2

Example 3 - data

<table>
<thead>
<tr>
<th>t</th>
<th>Revenue</th>
<th>Labor exp.</th>
<th>Material exp.</th>
<th>Optg. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30,000</td>
<td>20,000</td>
<td>4,000</td>
<td>6,000</td>
</tr>
<tr>
<td>2</td>
<td>35,000</td>
<td>22,000</td>
<td>6,000</td>
<td>7,000</td>
</tr>
<tr>
<td>3</td>
<td>35,000</td>
<td>22,000</td>
<td>5,500</td>
<td>7,500</td>
</tr>
<tr>
<td>4</td>
<td>35,000</td>
<td>20,000</td>
<td>7,200</td>
<td>7,800</td>
</tr>
<tr>
<td>5</td>
<td>35,000</td>
<td>20,000</td>
<td>7,000</td>
<td>8,000</td>
</tr>
<tr>
<td>6</td>
<td>18,000</td>
<td>10,000</td>
<td>3,000</td>
<td>5,000</td>
</tr>
</tbody>
</table>

Values not adjusted for inflation.

Example 3 - unadjusted

<table>
<thead>
<tr>
<th>t</th>
<th>Optg. Cash</th>
<th>Eqpt. Dep.</th>
<th>Tax. Inc.</th>
<th>35% Profit</th>
<th>Cash A/T</th>
<th>Cash A/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6,000</td>
<td>1,000</td>
<td>5,000</td>
<td>1,750</td>
<td>3,250</td>
<td>4,250</td>
</tr>
<tr>
<td>2</td>
<td>7,000</td>
<td>2,000</td>
<td>5,000</td>
<td>1,750</td>
<td>3,250</td>
<td>5,250</td>
</tr>
<tr>
<td>3</td>
<td>7,500</td>
<td>2,000</td>
<td>5,500</td>
<td>1,925</td>
<td>3,575</td>
<td>5,575</td>
</tr>
<tr>
<td>4</td>
<td>7,800</td>
<td>2,000</td>
<td>5,800</td>
<td>2,030</td>
<td>3,770</td>
<td>5,770</td>
</tr>
<tr>
<td>5</td>
<td>8,000</td>
<td>2,000</td>
<td>6,000</td>
<td>2,100</td>
<td>3,900</td>
<td>5,900</td>
</tr>
<tr>
<td>6</td>
<td>5,000</td>
<td>1,000</td>
<td>4,000</td>
<td>1,400</td>
<td>2,600</td>
<td>3,600</td>
</tr>
</tbody>
</table>

Evaluate Cash A/T: PW(20%) = $6,773

Example 3 - adjusted data

<table>
<thead>
<tr>
<th>t</th>
<th>Revenue</th>
<th>Labor exp.</th>
<th>Material exp.</th>
<th>Optg. Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>32,100</td>
<td>22,000</td>
<td>4,120</td>
<td>5,980</td>
</tr>
<tr>
<td>2</td>
<td>40,072</td>
<td>26,620</td>
<td>6,365</td>
<td>7,086</td>
</tr>
<tr>
<td>3</td>
<td>42,877</td>
<td>29,282</td>
<td>6,010</td>
<td>7,585</td>
</tr>
<tr>
<td>4</td>
<td>45,878</td>
<td>29,282</td>
<td>8,104</td>
<td>8,492</td>
</tr>
<tr>
<td>5</td>
<td>49,089</td>
<td>32,210</td>
<td>8,115</td>
<td>8,764</td>
</tr>
<tr>
<td>6</td>
<td>27,013</td>
<td>17,716</td>
<td>3,582</td>
<td>5,715</td>
</tr>
</tbody>
</table>

f = 7%  f = 10%  f = 3%

Example 3 - adjusted

<table>
<thead>
<tr>
<th>t</th>
<th>Optg. Cash</th>
<th>Eqpt. Dep.</th>
<th>Tax. Inc.</th>
<th>35% Profit</th>
<th>Cash A/T</th>
<th>Cash A/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5,980</td>
<td>1,000</td>
<td>4,980</td>
<td>1,743</td>
<td>3,237</td>
<td>4,237</td>
</tr>
<tr>
<td>2</td>
<td>7,086</td>
<td>2,000</td>
<td>5,086</td>
<td>1,780</td>
<td>3,306</td>
<td>5,306</td>
</tr>
<tr>
<td>3</td>
<td>7,585</td>
<td>2,000</td>
<td>5,585</td>
<td>1,955</td>
<td>3,630</td>
<td>5,630</td>
</tr>
<tr>
<td>4</td>
<td>8,492</td>
<td>2,000</td>
<td>6,492</td>
<td>2,272</td>
<td>4,220</td>
<td>6,220</td>
</tr>
<tr>
<td>5</td>
<td>8,764</td>
<td>2,000</td>
<td>6,764</td>
<td>2,367</td>
<td>4,397</td>
<td>6,397</td>
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<tr>
<td>6</td>
<td>5,715</td>
<td>1,000</td>
<td>4,715</td>
<td>1,650</td>
<td>3,065</td>
<td>4,065</td>
</tr>
</tbody>
</table>

Evaluate Cash A/T, i = (1.20)(1.03) – 1

PW(23.6%) = $5,905 < $6,773

Observations

- The lower PW when considering inflation is caused by the high labor inflation rate compared to the other rates.
- Depreciation amounts are not adjusted for inflation (in U.S.).
- Loan interest rates may or may not be adjusted periodically to compensate for inflation.

Summary

- Different approaches to inflation may yield different cash flow patterns.
- Effects of differential inflation rates may not be intuitive.
- Spreadsheets are almost a necessity for more involved problems.

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Uncertainty - Part 1

Overview

- Definitions, concepts
- Scenario analysis
- Expected value analysis
- Sensitivity analysis
- Examples

Some definitions

- Uncertainty - lack of knowledge about the relative chances or probabilities of events that may or may not occur
- Risk - a quantifiable measure of the chance or probability of an undesirable outcome (sometimes a function of the chance or probability)

Scenario analysis

The formulation of several outcomes that may occur, and the subsequent analysis for each outcome.

May be used with any one of the criteria, such as FW, PW, AW, ROR, BCR.

Example 1

A project requires an investment of $200,000, and is expected to produce annual net cash flows of $70,000 for 8 years in the most likely outlook.

However, if the economic situation is poor (pessimistic outlook), then the net cash flow would be only $50,000 for 7 years.

Example 1, cont.

If the economic situation is very good (optimistic outlook), then the net cash flow would be $85,000 for 9 years.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>N</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pessimistic</td>
<td>7</td>
<td>50,000</td>
</tr>
<tr>
<td>Most likely</td>
<td>8</td>
<td>70,000</td>
</tr>
<tr>
<td>Optimistic</td>
<td>9</td>
<td>85,000</td>
</tr>
</tbody>
</table>
If MARR = 20%, we can evaluate each scenario using PW(20%):

\[
(P/A, N, 20\%) \quad B(20\%)
\]

\[
(3.6046)(50,000) = 180,230 \\
(3.8372)(70,000) = 268,601 \\
(4.0310)(85,000) = 342,632
\]

\[
PW_{p}(20\%) = 180,230 - 200,000 = -19,770
\]

\[
PW_{m}(20\%) = 68,601
\]

\[
PW_{o}(20\%) = 142,632
\]

In this example the numerical data is the same as in Ex. 1, but we are willing to assign probability values to the three outcomes:

- Pessimistic: 0.20
- Most likely: 0.50
- Optimistic: 0.30

\[
\sum = 1.0
\]

Expected PW(20%) = 

\[
(0.2)(-19,770) + (0.5)(68,601) + (0.3)(142,632)
\]

\[
= $73,136 \Rightarrow Appears \ favorable
\]

Expected value analysis is a reasonable approach if the firm can tolerate the risks of losses.

Works with

- PW (if revenues known)
- FW, AW (if N is same)

Generally does not work with

- ROR (relative measure)
- BCR (relative measure)

An employer is considering two options for employee auto use:

- Buy a car and pay all costs
- Reimburse employees $0.30/mile for business use of their private cars.

Data:

- Purchase cost: $25,000
- Estimated resale after 5 years of use: $5,000
- Insurance/year: $1,200
- License, tax/year: $400

More data:

- Repair/year: $800
- Fuel: $0.05/mile
- Oil: $25/5,000 miles
- Tires: $600/40,000 miles

With MARR = 12%, under what circumstances should the employer buy a car for the employee(s).

Approach: Separate the costs into fixed costs that do not depend on miles driven for business use, and variable costs that do.

Loss in value:

\[
(25,000)(A/P, 12\%, 5) - 5,000(A/F, 12\%, 5)
\]

\[
= (25,000)(0.2774) - 5,000(0.1574) \\
= 6,935 - 787 = 6,148/year
\]

Other fixed costs/year:

\[
1,200 + 400 + 800 = 2,400/year
\]

Total fixed costs/year:

\[
6,148 + 2,400 = 8,548/year
\]

Variable costs:

- Fuel: 0.05/mile,
- Oil: $25/5,000 miles = 0.005/mile
- Tires: $600/40,000 miles = 0.015/mile

Total variable costs: $0.07/mile
Example 3, equation

We can formulate an equation to find the Breakeven point:

Let \( X \) = miles driven for business use by employee(s)

Then if we solve for \( X \) in

\[ 8,548 + 0.07X = 0.30X \]

We will obtain the Breakeven point

Here \( X = 37,165 \)

Example 3, breakeven graph

Breakeven analysis

Business owns vehicle

Employee reimbursement

miles/year

$-5,000 $10,000 $15,000

0 10,000 20,000 30,000 40,000 50,000
cost

Uncertainty - Part 1

Summary

- Definitions: Uncertainty, risk
- Scenario analysis: Pessimistic, most likely, optimistic outlooks
- Expected PW analysis
- Sensitivity analysis, 1 parameter: Breakeven analysis

ISyE 3025
Engineering Economy

Uncertainty
Part 2

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Georgia Institute of Technology

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Uncertainty - Part 2

Overview

Sensitivity analysis example with three parameters
- Retirement savings plan
- Parameters
  - Real interest rate
  - Real salary growth rate
  - Years in the plan

Example: Goal = $1 million

We would like to accumulate an amount at retirement equal to $1 million in constant dollars.

Method 1: Deposit the same amount each year in constant dollars. Vary the assumptions about real interest rates.

Assume general inflation \( f = 3\% \), hold \( f \) constant.
Uncertainty - Part 2

Method 1: $i' = 3\%, \ g = 0$

Work in constant dollars:

\[
A = F(A/F, i, N) = F(A/F, 3\%, 30)
\]

\[
A = 1,000,000(0.021019) = 21,019 \text{ not feasible?}
\]

Method 1: $i' = 8\%, \ g = 0$

Assume real interest rate $= 8\%$.

Work in constant dollars again:

\[
A = F(A/F, i, N) = F(A/F, 8\%, 30)
\]

\[
A = 1,000,000(0.008827) = 8,827
\]

Most likely not feasible for someone just starting a career.

Method 2

- Save $5,000 per year.
- Increase this amount according to real growth in salary $g \geq 0$
- Real interest rate is at least 3\% per year above inflation:

\[
(1 + i') = \frac{(1 + i)}{(1 + f)}
\]

where:
- $i$ = market, or actual rate
- $f$ = inflation rate
- $i'$ = “real” interest rate

Deposit Inflation Deposit Interest Value at horizon

<table>
<thead>
<tr>
<th>t</th>
<th>Deposit const. $</th>
<th>Inflation factor</th>
<th>Deposit actual $</th>
<th>Interest factor</th>
<th>Value at horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,000</td>
<td>1.0300</td>
<td>5,150</td>
<td>5.534</td>
<td>28,600</td>
</tr>
<tr>
<td>2</td>
<td>5,000</td>
<td>1.0609</td>
<td>5,305</td>
<td>5.2346</td>
<td>27,767</td>
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<tr>
<td>3</td>
<td>5,000</td>
<td>1.0927</td>
<td>5,464</td>
<td>4.9341</td>
<td>26,958</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
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<td>28</td>
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<td>1.1255</td>
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<td>11,783</td>
<td>1.0609</td>
<td>12,500</td>
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<tr>
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<td>2.4273</td>
<td>12,136</td>
<td>1.0000</td>
<td>12,136</td>
</tr>
</tbody>
</table>

\[
\text{\$577,390}(1+0.03)^{30} = \text{\$237,877} \text{ constant $}
\]

\[
\text{\$577,390} \text{ actual $}
\]

Method 2: $i' = 3\%, \ g = 0$

Work in constant dollars again:

\[
P = A_t\left(P/A, i, g, N\right)
\]

\[
P = A_t\left(1-(1+g)^N\right)/\left(1-i\right)
\]

\[
A_t = 5,000(1.02)^1 = 5,100
\]

\[
P = 5,000\left(1- (1.02)^{30}(1.06)^{-30}\right)
\]

\[
(0.06 – 0.02)
\]

\[
= 87,286 \text{ constant $}
\]

\[
F = 87,286(1.06)^{30} = 501,347 \text{ constant $}
\]

\[
F = 501,347(1.03)^{30} = 1,216,900 \text{ actual $}
\]

This may be a most likely scenario.
**Method 2: \( i' = 6\%, \ g = 2\% \)**

<table>
<thead>
<tr>
<th>Deposit</th>
<th>Inflation factor</th>
<th>Deposit actual $</th>
<th>Interest factor</th>
<th>Value at horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,100</td>
<td>1.0300</td>
<td>5,253</td>
<td>12.7688</td>
</tr>
<tr>
<td>2</td>
<td>5,202</td>
<td>1.0609</td>
<td>5,519</td>
<td>11.6952</td>
</tr>
<tr>
<td>3</td>
<td>5,306</td>
<td>1.0927</td>
<td>5,798</td>
<td>10.7118</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>28</td>
<td>8,705</td>
<td>2.2879</td>
<td>19,917</td>
<td>1.1920</td>
</tr>
<tr>
<td>29</td>
<td>8,879</td>
<td>2.3566</td>
<td>20,924</td>
<td>1.0918</td>
</tr>
<tr>
<td>30</td>
<td>9,057</td>
<td>2.4273</td>
<td>21,983</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

\$1,216,900 \times (1+0.03)^{-30} = \$501,347

**Constant $ accumulation (000)**

---

**Method 2: \( i' = 8\%, \ g = 5\% \)**

Work in constant dollars again:

\[ P = A_i \frac{P}{A_i}(i', g, N) \]

\[ = A_i \left[ \frac{1}{1 + (1+i')^N} \right] \]

\[ A_i = 5,000(1.05)^1 = \$ 5,250 \]

\[ P = 5,250 \left[ \frac{1}{1 + (1.05)^{30}(1.08)^{-30}} \right] \]

\[ = \$ 99,837 \text{ constant $} \]

\[ F = 99,837(1.08)^{30} = \$ 1,004,625 \text{ const.$} \]

\[ F = 1,004,625(1.03)^{30} = \$ 2,438,489 \text{ act.$} \]

This may be an optimistic scenario.

---

**Method 2: \( i' = 8\%, \ g = 5\% \)**

<table>
<thead>
<tr>
<th>Deposit</th>
<th>Inflation factor</th>
<th>Deposit actual $</th>
<th>Interest factor</th>
<th>Value at horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,250</td>
<td>1.0300</td>
<td>5,408</td>
<td>21.9568</td>
</tr>
<tr>
<td>2</td>
<td>5,513</td>
<td>1.0609</td>
<td>5,848</td>
<td>19.7382</td>
</tr>
<tr>
<td>3</td>
<td>5,788</td>
<td>1.0927</td>
<td>6,325</td>
<td>17.7438</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>28</td>
<td>19,061</td>
<td>2.2879</td>
<td>44,845</td>
<td>1.2374</td>
</tr>
<tr>
<td>29</td>
<td>20,581</td>
<td>2.3566</td>
<td>48,500</td>
<td>1.1124</td>
</tr>
<tr>
<td>30</td>
<td>21,610</td>
<td>2.4273</td>
<td>52,452</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

\$2,438,489 \times (1+0.03)^{-30} = \$1,004,625

**Constant $ accumulation (000)**

---

**Summary chart, \( N = 30 \)**

**Summary table, \( N = 30 \)**

<table>
<thead>
<tr>
<th>( i', \ g )</th>
<th>0%</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td>238</td>
<td>273</td>
<td>314</td>
<td>364</td>
<td>424</td>
<td>497</td>
</tr>
<tr>
<td>4%</td>
<td>280</td>
<td>319</td>
<td>365</td>
<td>420</td>
<td>487</td>
<td>566</td>
</tr>
<tr>
<td>5%</td>
<td>332</td>
<td>375</td>
<td>427</td>
<td>488</td>
<td>561</td>
<td>648</td>
</tr>
<tr>
<td>6%</td>
<td>395</td>
<td>444</td>
<td>501</td>
<td>569</td>
<td>650</td>
<td>746</td>
</tr>
<tr>
<td>7%</td>
<td>472</td>
<td>527</td>
<td>592</td>
<td>668</td>
<td>757</td>
<td>864</td>
</tr>
<tr>
<td>8%</td>
<td>566</td>
<td>629</td>
<td>701</td>
<td>786</td>
<td>887</td>
<td>1,005</td>
</tr>
</tbody>
</table>

**Constant $ accumulation (000)**

---

**Summary chart, \( N = 40 \)**
Uncertainty - Part 2

Summary table, $N = 40$

<table>
<thead>
<tr>
<th>$i'$, g</th>
<th>0%</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td>377</td>
<td>448</td>
<td>538</td>
<td>652</td>
<td>800</td>
<td>992</td>
</tr>
<tr>
<td>4%</td>
<td>475</td>
<td>558</td>
<td>661</td>
<td>793</td>
<td>960</td>
<td>1,175</td>
</tr>
<tr>
<td>5%</td>
<td>604</td>
<td>701</td>
<td>821</td>
<td>973</td>
<td>1,164</td>
<td>1,408</td>
</tr>
<tr>
<td>6%</td>
<td>774</td>
<td>888</td>
<td>1,030</td>
<td>1,206</td>
<td>1,426</td>
<td>1,704</td>
</tr>
<tr>
<td>7%</td>
<td>998</td>
<td>1,135</td>
<td>1,302</td>
<td>1,508</td>
<td>1,763</td>
<td>2,083</td>
</tr>
<tr>
<td>8%</td>
<td>1,295</td>
<td>1,460</td>
<td>1,659</td>
<td>1,902</td>
<td>2,200</td>
<td>2,570</td>
</tr>
</tbody>
</table>

Constant $\$ accumulation (000)$

Uncertainty - Part 2

Summary

- Sensitivity analysis of retirement savings plan
  - Real interest rate, $i'$$
  - Real salary growth rate, $g$
  - Years in the plan, $N$
  - Inflation held constant
- It is possible to accumulate $\$ 1 million with compounding
- Actual analysis done with spreadsheets