Basic Concepts

Course Focus

This course is focused on the principles and procedures for making sound economic decisions.

The Jackpot!

“1 million jackpot turns sour: Lotto winner kept from selling part of prize -- and sues”  (Newspaper headline)

Award: $50,000/yr for 20 yrs ($36,000/yr after taxes)
Deal: Company pays $241,000 now (after taxes) for next 10 yrs payments ($500,000 before taxes)

Pay Now or Pay Later

An automobile insurance company offers customers the opportunity to either pay for 6 months insurance in full now or pay half now and half in 60 days. The second option requires a $5 service fee due now. If the premium was $150, would you take them up on their offer?

“Have we got a deal for you!”

The Cost of Paying Cash

<table>
<thead>
<tr>
<th>Investment</th>
<th>$10,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Mkt Rate</td>
<td>6.5%</td>
</tr>
<tr>
<td>Investment Period</td>
<td>48 mo.</td>
</tr>
<tr>
<td>Interest Earned</td>
<td>$2,960.20</td>
</tr>
</tbody>
</table>

The Cost of Financing

<table>
<thead>
<tr>
<th>Amt Financed</th>
<th>$10,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.P.R.</td>
<td>10.5%</td>
</tr>
<tr>
<td>Term of Loan</td>
<td>48 mo.</td>
</tr>
<tr>
<td>Interest Paid</td>
<td>$2,289.44</td>
</tr>
</tbody>
</table>

“You save $670.76 to finance your car than pay cash!”
For you to be able to make wise economic decisions. To be proficient in comparing alternatives, able to see and assess tradeoffs, able to evaluate justifications prepared by others, comfortable using financial language, glad you took this course!

Alternatives: a choice among two or more things. Some choices obvious, some not. 1 possible choice: “do nothing”


All that is common is irrelevant to the decision (which also means that the past is irrelevant, except as a guide to predict future events, i.e., it is a “sunk cost”)

Both monetary and non-monetary.

Course focus
3 illustrative problems
Course objectives
4 basic principles
The Concept of Equivalence
(“Time Value of Money”)

Concept of Equivalence

Overview

- Compound interest & capital growth
- “Time Value of Money”

Concept of Equivalence

Two viewpoints of “interest”

1. Borrower’s viewpoint:
   Money paid for use of borrowed funds

2. Investor’s viewpoint:
   Return, or capital growth, from the productive investment of capital

Interest rate:

\( i = \) Amount accrued/unit time

Concept of Equivalence

Compound interest: An example

What is the capital growth of $100.00 invested at \( i = 3\% \) per year for 3 years?

<table>
<thead>
<tr>
<th>Capital at ( t )</th>
<th>Interest from ( t ) to ( t+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.00</td>
<td>0.03(100.00) = $3.00</td>
</tr>
<tr>
<td>$103.00</td>
<td>0.03(103.00) = $3.09</td>
</tr>
<tr>
<td>$106.09</td>
<td>0.03(106.09) = $3.18</td>
</tr>
<tr>
<td>$109.27</td>
<td></td>
</tr>
</tbody>
</table>

Concept of Equivalence

Compound interest: A formula

\[
\text{Capital Interest from } t \text{ at } t \text{ to } t+1 = P(1+i)^n \]

\[
P = P(1+i) \quad P + iP = P(1+i) \\
2P(1+i) + iP(1+i) = P(1+i)^2 \\
3P(1+i)^2 + iP(1+i)^2 = P(1+i)^3 \\
N = P(1+i)^n
\]

Concept of Equivalence

Our previous example . . .

What is the capital growth of $100.00 invested at \( i = 3\% \) per year for 3 years?

\[100.00(1.03)^3 = \$109.27\]
**Concept of Equivalence**

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**“Time Value of Money:” Example**

![Timeline with $100]  
At $i = 0.05/yr$:

\[ F = P(1+i)^N \]

\[ = 100(1.05)^5 \]

\[ = 127.63 \]

---

**Are these sums equivalent?**

![Timeline with $100 and $120]  
Equivalent at $i = 0.03/yr$?

\[ F = 100(1.03)^5 = 115.93 \]

$100 at t = 0 is not equivalent to $120 at t = 5. Prefer $120 five years from today.

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**Terminology: “Compounding”**

Calculating an equivalent amount money in the future amount given some amount of money in the present

---

**Another example...**

![Timeline with $500]  
At $i = 0.06/yr$:

\[ F = P(1+i)^N \]

\[ = 500(1.06)^{-10} \]

\[ = 279.20 \]

---

**Are these sums equivalent?**

![Timeline with $300 and $500]  
$300 at t = 0 is not equivalent to $500 at t = 11 when $i = 0.06/yr$. Prefer $300 today.

---

**Terminology: “Discounting”**

Calculating an equivalent amount money in the present amount given some amount of money in the future
Concept of Equivalence

One more example . . . Find \( F \)

\[
F = 50(1.1)^3 + 50(1.1)^2 + 50(1.1)^1 + 50 = \$232.05
\]

Concept of Equivalence

One more example . . . Find \( P \)

\[
P = 50(1.1)^{-1} + 50(1.1)^{-2} + 50(1.1)^{-3} + 50(1.1)^{-4} = \$158.49
\]

Concept of Equivalence

\( F \) and \( P \) equivalent?

Is \$158.49 at \( t = 0 \) equivalent to \$232.05 at \( t = 4? \)

Yes! \( 158.49(1.1)^4 = \$232.05 \)

Concept of Equivalence

Summary

- Compound interest & capital growth
  - Two views of “interest”
  - \( F = P(1+i)^N \)

Concept of Equivalence

Summary

- “Time Value of Money”
  - 3 examples: compounding, discounting, both
  - Equivalence depends on:
    1 - amount of money;
    2 - their timing; and
    3 - an interest rate
Equivalence Formulas

Overview

- **Purpose**
- 4 classic equivalence models
  - Single cash flow
  - Uniform cash flow series
  - Arithmetic gradient cash flow series
  - Geometric gradient cash flow series

To facilitate performing equivalence calculations

A cash flow convention

Several cash flow conventions exist: *this course will use end-of-period models*

$p = 10,000$

Time $T_0$, $T_1$, $T_2$, $T_3$... $T_{N-1}$ $T_N$

$Yr$, Qtr Yr, Mo, Wk, Day, etc

"End-of-Year Convention"

Equivalence Formulas

Four classic equivalence models

- Single cash flow
- Uniform series
- Arithmetic gradient series
- Geometric gradient series

Single cash flow: The model

$p = ?$

$T_0$, $T_1$, $T_2$, $T_3$... $T_{N-1}$ $T_N$

$T_0$ = past, present, or future

$T_N$ = $N$ periods after $T_0$

$F = p(1+i)^{T_N - T_0}$

$F = p(1+i)^N$

$F = p(F/P, i, N)$

Appendix

Single cash flow: An example

$F = ?$

$p = 100$

$3, 4, 5, 6, 7, 8$

What future amount $F$ at time $t = 8$ is equivalent to 100 at $t = 3$ if $i = 0.05$/year?

$F = 100(F/P, 0.05, 5)$

$= 100(1.276)$

$= 127.60$
**Single cash flow: The model**

\[ P = ? \]

\[ F = ? \]

\[ P = F(1+i)^{T_0-T_N} \]

\[ = F(1+i)^N \]

\[ = F(P/F, i, N) \]

**Appendix**

What amount \( P \) at \( t = -3 \) is equivalent to 500 at \( t = 7 \) if \( i = 0.06/\text{year} \)?

\[ P = 500(P/F, 0.06, 10) \]

\[ = 500(0.5584) \]

\[ = 279.20 \]

**Uniform cash flow series: The model**

\[ F = ? \]

\[ A = ? \]

\[ P = ? \]

\[ F = A(1+i)^{N-1}+A(1+i)^{N-2}+\ldots+A(1+i)+A \]

\[ F(1+i)^{-1} = A[(1+i)^N-1]/i \]

\[ F = A[(1+i)^N-1]/i \]

\[ = A(F/A, i, N) \]

\[ A = P[i/(1+i)^N-1] \]

\[ = A(F/A, i, N) \]

**Uniform cash flow series: Example**

What future amount \( F \) at \( t = 4 \) is equivalent to $50 each year for the next four years if \( i = 0.1/\text{year} \)?

\[ F = 50(F/A, 0.1, 4) \]

\[ = 50(4.641) \]

\[ = 232.05 \]
**Uniform cash flow series: Example**

\[
\begin{align*}
P & = \? \\
\text{"P"} &= 100(P/A, 0.12, 3) \\
&= 100(2.4020) \\
&= 240.20 \\
P & = 240(P/F, 0.12, 7) \\
&= 108.64
\end{align*}
\]

**Arithmetic gradient series: The model**

\[
\begin{align*}
P & = G\left[\frac{(1+i)^N-iN-1}{i} \right] \\
&= G(P/G, i, N)
\end{align*}
\]

**An example**

Maintenance costs

\[
P = \? \\
P = -3000(P/A, 0.08, 4) \\
&= -1000(P/G, 0.08, 4) \\
&= -14,586 \\
A & = -14,586(A/P, 0.08, 4) = -4404
\]

**An example: Another way**

Maintenance costs

\[
A = \? \\
A = -3000 -1000(A/G, 0.08, 4) \\
&= -4404
\]

**Geometric gradient series: The model**

\[
P = A[1/(1+i) + A(1+g)/(1+i) \ldots + A(1+g)^{N-1}/(1+i)^N] \\
P = A_1/N(1+i)^{-1} \text{ for } i \neq g \\
P = A_1(P/A_1,i,g,N)
\]
An example

Assume you plan to save 10% of your salary each year and invest it in an account at 8%/year, how much would you accumulate in the account in 10 years?

Assume your current salary is $60,000, and raises are expected at the rate of 6%/year.

Equivalence Formulas

Summary

- Purpose
- 4 classic equivalence models
  - Single cash flow \((F/P,i,n)\) & \((P/F,i,n)\)
  - Uniform cash flow series
    \((F/A,i,n)\), \((P/A,i,n)\), \((A/F,i,n)\), \((A/P,i,n)\)

Interest Rates

Overview

- Time scale conversions
- “Nominal” versus “Effective” rates
Interest Rates

Time scale conversion: Example
If you were offered \( i_4 = 0.04 \) per quarter year, what semi-annual rate, \( i_2 \), would make you indifferent between \( i_4 = 0.04 \) and \( i_2 \)?

\[
F = P(1+i_4)^4 = P(1+i_2)^2 \\
= (1.04)^4 - 1 \\
= 0.0816
\]

Interest Rates

Time scale conversion: Formula

\[
P = F(1+i_2)^{M1} = P(1+i_{M2})^{M2} \\
i_{M1} = (1+i_{M2})^{M2/M1} - 1
\]

Interest Rates

Nominal vs. Effective Rates

Nominal Annual Interest Rate, \( r = M_i \)

Credit cards often charge 1.5%/mo for unpaid balances and report it as “18% compounded monthly,” i.e., \( 12(0.015) = 0.18 \).

Interest Rates

Nominal vs. Effective Rates

Effective Annual Interest Rate:

\[
i = (1+i_4)^{M-1} \\
= (1.015)^{12} - 1 \\
= 0.196
\]

Another view of “effective” interest

A bank offers loans of $2500 for 30 months at an advertised interest rate of 2%/mo. The monthly payments are computed as follows:

Interest = 0.02(2500)30 = $1500; Processing Fee = $50;
Total Owed = 2500+1500+50 = $4050; Monthly Payment = 4050/30 = $135. What is the effective annual interest rate?
Solution

Interest Rates

\[ 2500 = 135(P/A, i_{12}, 30) \]

\[ i_{12} = 0.036 \text{ per month} \]

\[ i = (1.036)^{12} - 1 = 0.529/\text{year} \]

Summary

Interest Rates

- Time scale conversions
  \[ i_{M1} = \frac{(1+i_{M2})^{M2/M1} - 1}{M1} \]

- Nominal vs effective interest rates
  \[ r = \frac{M_i}{M} \]
  \[ i = \frac{(1+i_{M})^{M} - 1}{M} \]

\[ i \text{ should include all costs} \]