ISyE 3025 Engineering Economy

Basic Concepts

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Basic Concepts

Overview 1

- Course focus
- 3 illustrative problems
- Course objectives
- 4 basic principles

Basic Concepts

Course Focus 2

This course is focused on the principles and procedures for making sound economic decisions

Basic Concepts

The Jackpot! 3

"1 million jackpot turns sour: Lotto winner kept from selling part of prize -- and sues" (Newspaper headline)

Award: $50,000/yr for 20 yrs ($36,000/yr after taxes)

Deal: Company pays $241,000 now (after taxes) for next 10 yrs payments ($500,000 before taxes)

Basic Concepts

Pay Now or Pay Later 4

An automobile insurance company offers customers the opportunity to either pay for 6 months insurance in full now or pay half now and half in 60 days. The second option requires a $5 service fee due now. If the premium was $150, would you take them up on their offer?

Basic Concepts

"Have we got a deal for you!" 5

The Cost of Paying Cash	
Investment	$10,000.00
Money Mkt Rate	6.5%
Investment Period	48 mo.
Interest Earned	$2,960.20

The Cost of Financing	
Amt Financed	$10,000.00
A.P.R.	10.5%
Term of Loan	48 mo.
Interest Paid	$2,289.44

"You save $670.76 to finance your car than pay cash!"

? ? ?

Basic Concepts

Course Objectives

6

For you to be able to make wise economic decisions . . . To be

- Proficient in comparing alternatives
- Able to see and assess tradeoffs
- Able to evaluate justifications prepared by others
- Comfortable using financial language
- Glad you took this course!

Basic Concepts

Four basic principles

7

1 - An economic decision is no better than the alternatives considered

Alternatives: a choice among two or more things

- Some choices obvious, some not
- 1 possible choice: "do nothing"

Basic Concepts

. . . Four basic principles

8

2 - An economic decision is no better than the forecasts describing each alternative

- Forecasting . . . *not easy!*
- Some aspects certain, others uncertain
- Monetary & Non-monetary
- Monetary: amounts & *timing*

Basic Concepts

. . . Four basic principles

9

3 - An economic decision should be based on the differences among alternatives

- All that is common is irrelevant to the decision
  (which also means that the past is irrelevant, except as a guide to predict future events, i.e., it is a "sunk cost")

Basic Concepts

. . . Four basic principles

10

4 - An economic decision should be based on the objective of making the "best" use of limited resources

- Both monetary and non-monetary

Basic Concepts

Summary

11

- Course focus
- 3 illustrative problems
- Course objectives
- 4 basic principles

ISyE 3025
Engineering
Economy

The Concept of Equivalence

(“Time Value of Money”)

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Concept of Equivalence

Overview (1)

- ◆ Compound interest & capital growth
- ◆ “Time Value of Money”

Concept of Equivalence

Two viewpoints of “interest” (2)

1 • **Borrower’s viewpoint:**
Money paid for use of borrowed funds

2 • **Investor’s viewpoint:**
Return, or capital growth, from the productive investment of capital

Interest rate:
i = Amount accrued/unit time

Concept of Equivalence

Compound interest: An example (3)

What is the capital growth of \$100.00 invested at i = 3% per year for 3 years?

t	Capital at t	Interest from t to t+1
0	\$100.00	0.03(100.00) = \$3.00
1	\$103.00	0.03(103.00) = \$3.09
2	\$106.09	0.03(106.09) = \$3.18
3	\$109.27	

Concept of Equivalence

Compound interest: A formula (4)

t	Capital at t	Interest from t to t+1
0	P	iP
1	$P+iP$	
	$= P(1+i)$	$iP(1+i)$
2	$P(1+i)+iP(1+i)$	
	$= P(1+i)^2$	$iP(1+i)^2$
3	$P(1+i)^2+iP(1+i)^2$	
	$= P(1+i)^3$	
N	$= P(1+i)^N$	

Concept of Equivalence

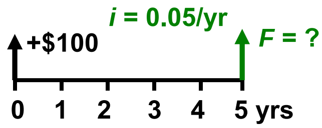
Our previous example . . . (5)

What is the capital growth of \$100.00 invested at i = 3% per year for 3 years?

$100.00(1.03)^3 = \$109.27$

Concept of Equivalence

“Time Value of Money:” Example 6



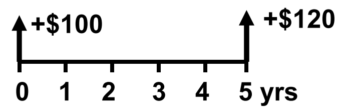
$F = P(1+i)^N$

$= \$100(1.05)^5$

$= \$127.63$

Concept of Equivalence

Are these sums equivalent? 7



Equivalent at $i = 0.03$/year?

$F = 100(1.03)^5 = \$115.93$

\$100 at $t = 0$ is not equivalent to \$120 at $t = 5$. Prefer \$120 five years from today.

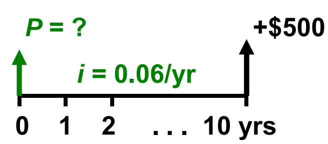
Concept of Equivalence

Terminology: “Compounding” 8

Calculating an equivalent amount money in the *future* amount given some amount of money in the *present*

Concept of Equivalence

Another example . . . 9



$F = P(1+i)^N$

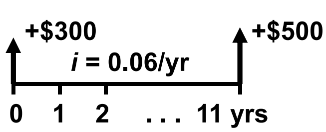
$P = F(1+i)^{-N}$

$P = 500(1.06)^{-10}$

$= \$279.20$

Concept of Equivalence

Are these sums equivalent? 10



$P = 500(1.06)^{-11} = \$263.39$

\$300 at $t = 0$ is not equivalent to \$500 at $t = 11$ when $i =$ 0.06/yr. Prefer \$300 today.

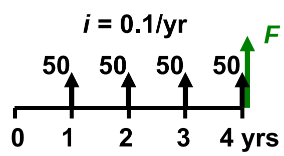
Concept of Equivalence

Terminology: “Discounting” 11

Calculating an equivalent amount money in the *present* amount given some amount of money in the *future*

Concept of Equivalence

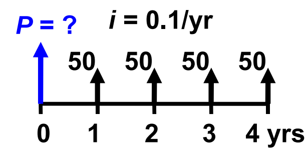
One more example . . . Find *F* — 12



$F = 50(1.1)^3 + 50(1.1)^2 + 50(1.1)^1 + 50 = \$232.05$

Concept of Equivalence

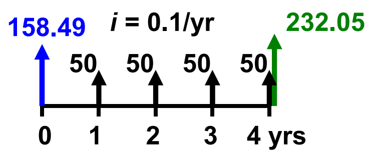
One more example . . . Find *P* — 13



$P = 50(1.1)^{-1} + 50(1.1)^{-2} + 50(1.1)^{-3} + 50(1.1)^{-4} = \$158.49$

Concept of Equivalence

F and *P* equivalent? — 14



Is \$158.49 at $t = 0$ equivalent to \$232.05 at $t = 4$?

Yes! $158.49(1.1)^4 = \$232.05$

Concept of Equivalence

Summary — 15

- Compound interest & capital growth
  - ➔ Two views of "interest"
  - ➔ $F = P(1+i)^N$

Concept of Equivalence

Summary — 16

- "Time Value of Money"
  - ➔ 3 examples: compounding, discounting, both
  - ➔ Equivalence depends on:
    1 - amount of money;
    2 - their timing; and
    3 - an interest rate

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Equivalence Formulas

(sometimes called Equivalence Factors)

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Equivalence Formulas

Overview — 1

- Purpose
- 4 classic equivalence models
  - ➔ Single cash flow
  - ➔ Uniform cash flow series
  - ➔ Arithmetic gradient cash flow series
  - ➔ Geometric gradient cash flow series

Equivalence Formulas

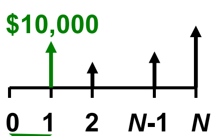
Purpose — 2

To facilitate performing equivalence calculations

Equivalence Formulas

A cash flow convention — 3

Several cash flow conventions exist: *this course will use end-of-period models*

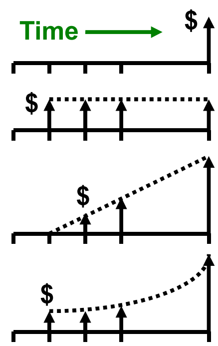


Yr, Qtr Yr, Mo, Wk, Day, etc

"End-of-Year Convention"

Equivalence Formulas

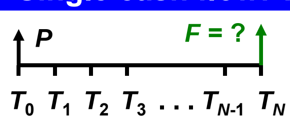
Four classic equivalence models — 4



- Single cash flow
- Uniform series
- Arithmetic gradient series
- Geometric gradient series

Equivalence Formulas

Single cash flow: The model — 5



T_0 = past, present, or future

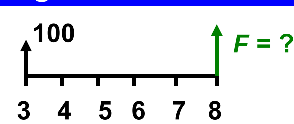
T_N = N periods after T_0

$$F = P(1+i)^{T_N - T_0}$$
$$= P(1+i)^N$$
$$= P(F/P,i,N)$$

Appendix

Equivalence Formulas

Single cash flow: An example — 6

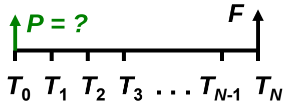


What future amount F at time $t = 8$ is equivalent to 100 at $t = 3$ if $i = 0.05$/year?

$$F = 100(F/P,0.05,5)$$
$$= 100(1.276)$$
$$= 127.60$$

Equivalence Formulas

Single cash flow: The model (7)



$P = F(1+i)^{T_0 - T_N}$

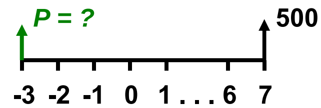
$= F(1+i)^{-N}$

$= F(P/F,i,N)$

Appendix

Equivalence Formulas

Single cash flow: Another example (8)



What amount P at $t = -3$ is equivalent to 500 at $t = 7$ if $i = 0.06$/year?

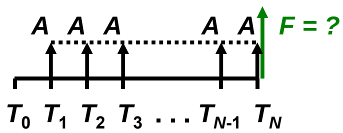
$P = 500(P/F,0.06,10)$

$= 500(0.5584)$

$= 279.20$

Equivalence Formulas

Uniform cash flow series: The model (9)



$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \ldots + A(1+i) + A$

↓ Algebra ↓

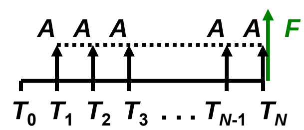
$F = A[(1+i)^{N-1} + (1+i)^{N-2} + \ldots + (1+i) + 1]\dfrac{[(1+i)-1]}{[(1+i)-1]}$

$F[(1+i)-1] = A[(1+i)^N + \cancel{(1+i)^{N-1}} + \cancel{(1+i)^{N-2}} + \ldots + \cancel{(1+i)}$
$\cancel{-(1+i)^{N-1}} \cancel{-(1+i)^{N-2}} - \ldots \cancel{-(1+i)} - 1]$

i

Equivalence Formulas

Uniform cash flow series: The model (10)



$F = A[\{(1+i)^N - 1\}/i]$

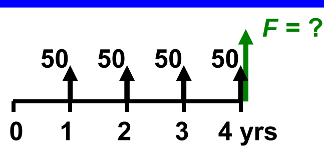
$= A(F/A,i,N)$

$A = F[i/\{(1+i)^N - 1\}]$

$= F(A/F,i,N)$

Equivalence Formulas

Uniform cash flow series: Example (11)



What future amount F at $t = 4$ is equivalent to \$50 each year for the next four years if $i = 0.1$/year?

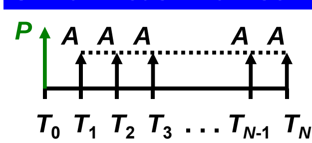
$F = 50(F/A,0.1,4)$

$= 50(4.641)$

$= 232.05$

Equivalence Formulas

Uniform cash flow series: The model (12)



$P = A(1+i)^{-1} + A(1+i)^{-2} + \ldots + A(1+i)^{-N}$

$P = A[\{(1+i)^N - 1\}/i(1+i)^N]$ Algebra

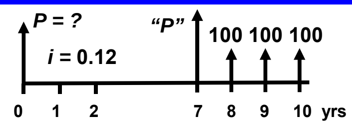
$= A(P/A,i,N)$

$A = P[i(1+i)^N/\{(1+i)^N - 1\}]$

$= P(A/P,i,N)$

Equivalence Formulas

Uniform cash flow series: Example 13

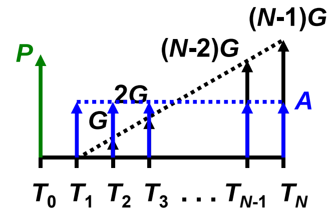


$"P" = 100(P/A, 0.12, 3)$
$= 100(2.4020)$
$= 240.20$

$P = 240(P/F, 0.12, 7)$
$= 108.64$

Equivalence Formulas

Arithmetic gradient series: The model 14



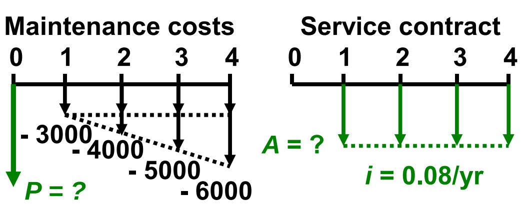
$P = G[\{(1+i)^N - iN - 1\}/[i^2(1+i)^N]$
$= G(P/G,i,N)$

$A = G[\{(1+i)^N - iN - 1\}/[i(1+i)^N - i\ ]$
$= G(A/G,i,N)$

Note correction

Equivalence Formulas

An example 15

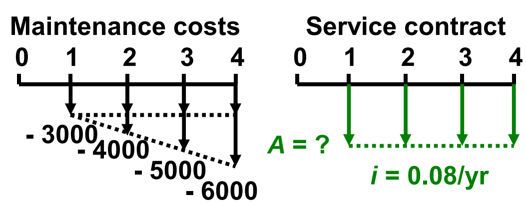


$P = -3000(P/A,.08,4)$
$\quad - 1000(P/G,.08,4)$
$= -14{,}586$

$A = -14{,}586(A/P,.08,4) = -4404$

Equivalence Formulas

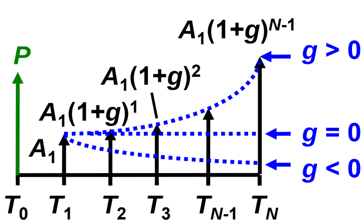
An example: Another way 16



$A = -3000 - 1000(A/G,.08,4)$
$= -4404$

Equivalence Formulas

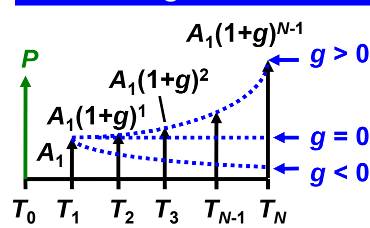
Geometric gradient series: The model 17



$P = [A_1/(1+i) + A_1(1+g)/(1+i) \ldots$
$\quad + A_1(1+g)^{N-1}/(1+i)^N]$

Equivalence Formulas

Geometric gradient series: The model 18



$$P = \begin{cases} A_1[\{1-(1+g)^N(1+i)^{-N}\}/\{i-g\}] & \text{for } i \neq g \\ A_1 N(1+i)^{-1} & \text{for } i = g \end{cases}$$

$= A_1(P/A_1,i,g,N)$

Equivalence Formulas

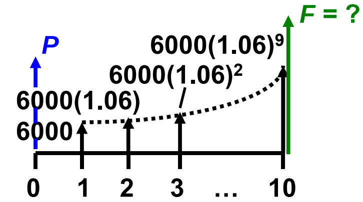
An example — 19

Assume you plan to save 10% of your salary each year and invest it in an account at 8%/year, how much would you accumulate in the account in 10 years?

Assume your current salary is $60,000, and raises are expected at the rate of 6%/year.

Equivalence Formulas

An example — 20



$P = 6000(P/A_1,.08,.06,10)$
$= 6000(8.5246)$
$= 51,148$

$F = 51,148(F/P,.08,10)$
$= 51,148(2.1589) = 110,425$ ← Note correction

Equivalence Formulas

Summary — 21

- Purpose
- 4 classic equivalence models
  - ➔ Single cash flow
    (*F/P,i,n*) & (*P/F,i,n*)
  - ➔ Uniform cash flow series
    (*F/A,i,n*), (*P/A,i,n*), (*A/F,i,n*), (*A/P,i,n*)

Equivalence Formulas

Summary — 22

- 4 classic equivalence models
  - ➔ Arithmetic gradient cash flow series
    (*P/G,i,n*)
  - ➔ Geometric gradient cash flow series
    ($P/A_1,i,g,n$)

ISyE 3025 Engineering Economy

Interest Rate Conversions

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Interest Rates

Overview — 1

- Time scale conversions
- “Nominal” versus “Effective” rates

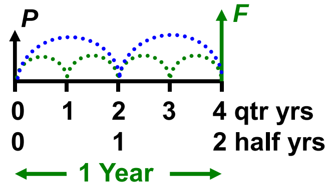
Interest Rates

Time scale conversion: Example 2

If you were offered $i_4 = 0.04$ per quarter year, what semi-annual rate, i_2, would make you indifferent between $i_4 = 0.04$ and i_2?

Interest Rates

Time scale conversion: Example 3



$F = P(1+i_4)^4 = P(1+i_2)^2$

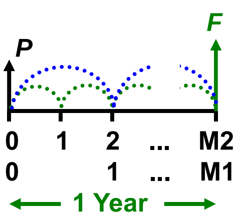
$i_2 = (1+i_4)^{4/2}-1$

$= (1.04)^2-1$

$= 0.0816$

Interest Rates

Time scale conversions: Formula 4



$F = P(1+i_{M1})^{M1} = P(1+i_{M2})^{M2}$

$i_{M1} = (1+i_{M2})^{M2/M1}-1$

Interest Rates

Nominal vs. Effective Rates 5

Nominal Annual Interest Rate, $r = Mi_M$

Credit cards often charge 1.5%/mo for unpaid balances and report it as "18% compounded monthly," i.e., 12(0.015) = 0.18.

Interest Rates

Nominal vs. Effective Rates 6

Effective Annual Interest Rate:

$i = (1+i_M)^M-1$

$= (1.015)^{12} - 1$

$= 0.196$

Interest Rates

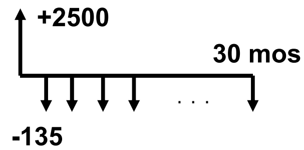
Another view of "effective" interest 7

A bank offers loans of $2500 for 30 months at an advertised interest rate of 2%/mo. The monthly payments are computed as follows: Interest = 0.02(2500)30 = $1500; Processing Fee = $50; Total Owed = 2500+1500+50 = $4050; Monthly Payment = 4050/30 = $135. What is the effective annual interest rate?

Interest Rates

Solution

8



$2500 = 135(P/A, i_{12}, 30)$

$i_{12} = 0.036$ per month

$i = (1.036)^{12} - 1 = 0.529$/year

Interest Rates

Summary

9

- Time scale conversions
  - ➔ $i_{M1} = (1+i_{M2})^{M2/M1}-1$
- Nominal vs effective interest rates
  - ➔ $r = Mi_M$
  - ➔ $i = (1+i_M)^M-1$
  - ➔ i should include all costs