Simulation of Conditional Value-at-Risk via Parallelism on Graphic Processing Units

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Outline of Topics

- Motivation
- Sequential procedure of CVaR via nested simulation
- Paralleled nested simulation with screening
- Experimental performance
- Conclusion & Question

Motivations

- Some simulation procedures take days, weeks or even months to run.
- Traditional supercomputers are inaccessible for most researchers and practitioners.
- New development of high performance computing catches out attention:
  - cloud computing: heterogeneous, massive storage, massive computing
  - parallel computing on GPUs: homogeneous, data light, moderate computing
- Characteristics of simulation in financial engineering: data security,moderate size computing.

GPU Computing

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Reported work shows

- 20% – 800% faster for sorting (key only)
- 30 – 50 times faster for random number generating
- almost 100 faster for Monte Carlo simulation of Black Shorel model

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How about its effectiveness on complex simulation such as nested simulation?

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- $\mathrm{VaR}_p = F_V^{-1}(p)$ is the p-quantile of F_V.
- $\mathrm{CVaR}_p = \int_p^1 F_V^{-1}(s)ds$.

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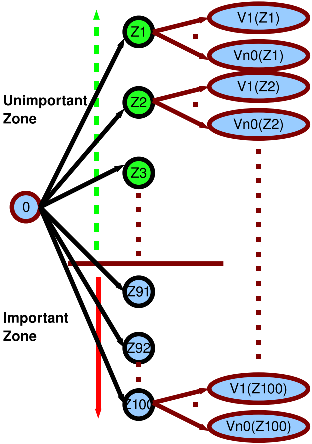
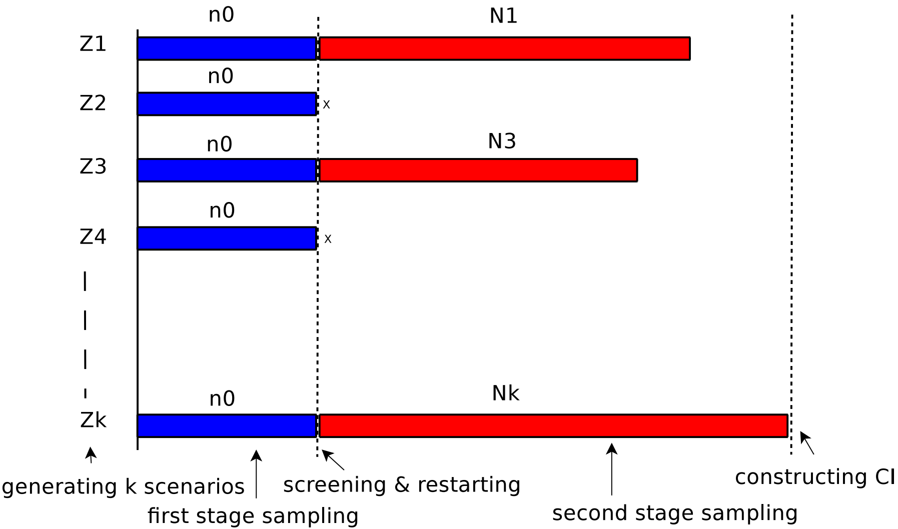
  underlying stock price $S_i(T), i = 1, \cdots, k$.
- inner level: estimating V conditionally on $Z(T)$ by Monte Carlo Simulation in risk neutral probability measure.
  generate the underlying stock prices
  $S(T \leq t \leq U)|S_i(T),\ i = 1, \cdots, k$

"Plain" Nested Simulation



- Simulate scenarios $Z_1, \cdots, Z_k$. Unknown true value $V_i = E[X|Z = Z_i]$.
- Simulate payoffs $X_{i1}, \cdots, X_{in_0}$ given Z_i. Estimate V_i by $(X_{i1} + \cdots + X_{in_0})/n_0$.

Nested Simulation with Screening



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- Restarting is used to avoid the selection bias inherited from screening.

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Experiments show 50-60 times faster than the sequential procedure.

Parallel Screening 1: Strategy

Sequential procedure:

- for any scenario $i = 1, \ldots, k$, compute at least $\lceil kp \rceil$ variances of difference $S_{ij}^2 = \sum_{l=1}^{n_0} \frac{(X_{il} - X_{jl})^2}{n_0 - 1} - \frac{\sum_{l=1}^{n_0}(X_{il} - X_{jl}) \sum_{l=1}^{n_0}(X_{il} - X_{jl})}{n_0(n_0 - 1)}$
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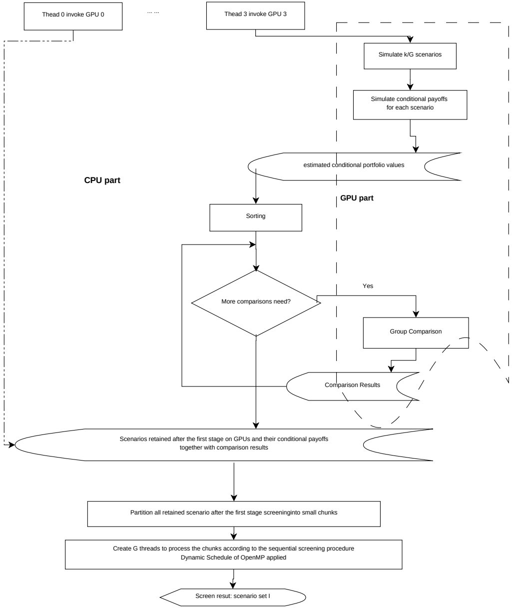
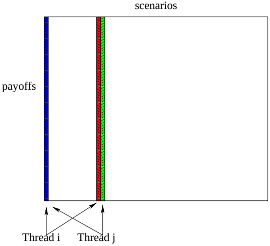
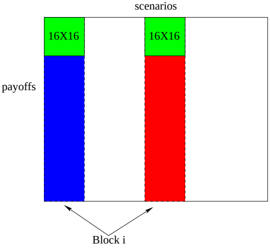
- randomly group the total k market scenario into G groups, each GPU processes one group.
- apply screening procedure within each group to eliminate some unimportant market scenarios.
- collect remaining scenarios from each GPU and perform screening again.

Parallel Screening 2: Flow Chart



Parallel Screening 3: Implementation on GPU





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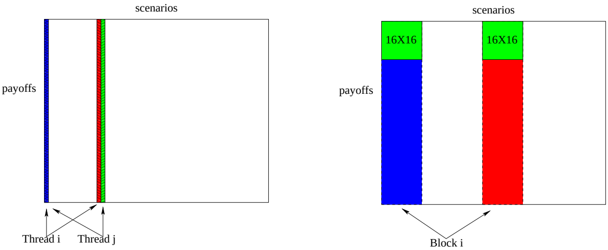
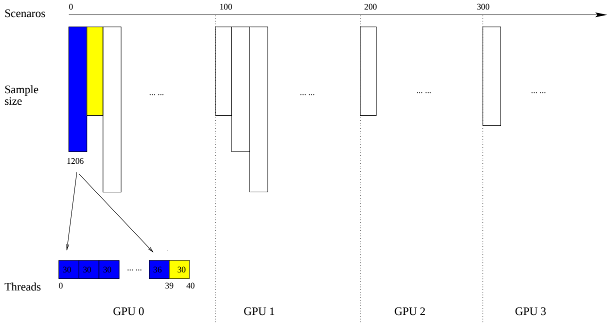


Scheme 1 is about 2 times faster than CPU only, scheme 2 is 30-50 times faster.

Second Stage Sampling



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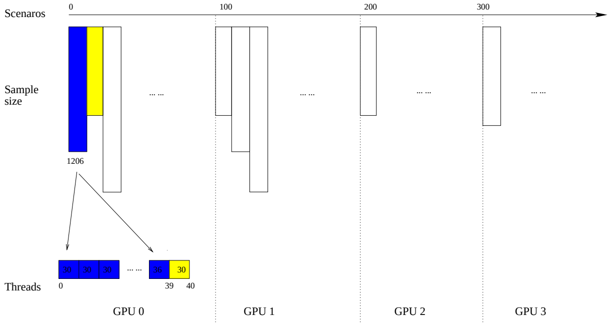
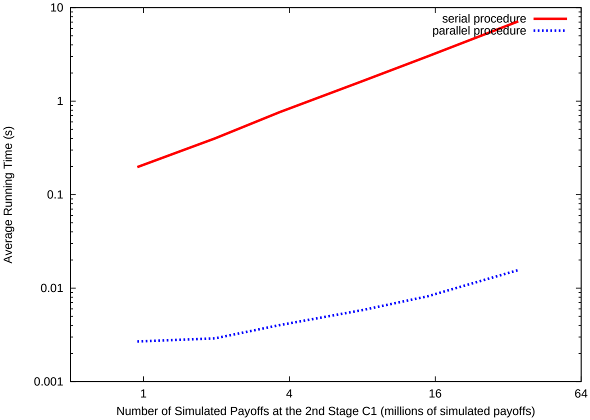


- balance the uneven load among different GPUs and threads
- experiments show 100-400 times faster

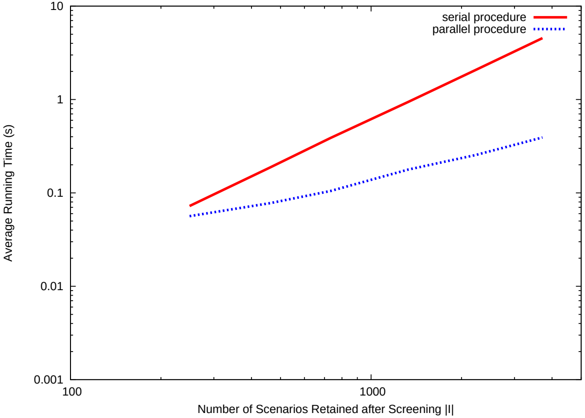
Experimental Setting

- hardware: one 3.0G Hz Core 2 Duo CPU, four GeForce 9800 GPUs, 6 GB memory.
- software: gcc 4.2.4 on Ubuntu 64 bits version.
- testing portfolios: shorting one put option and a portfolio with 8 options.
- number of replications: 100 and more.

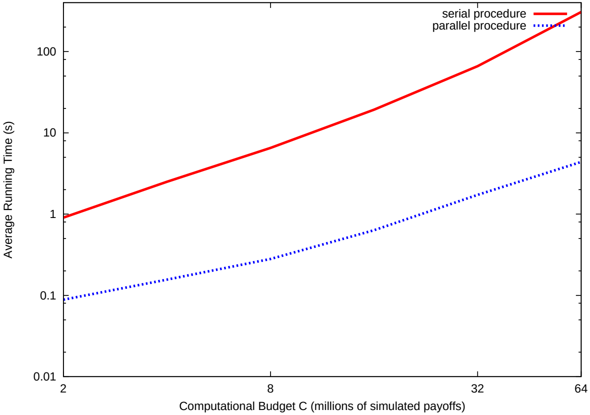
Speedup of Sampling



Speedup of Computing the CI



Speedup in Running Time



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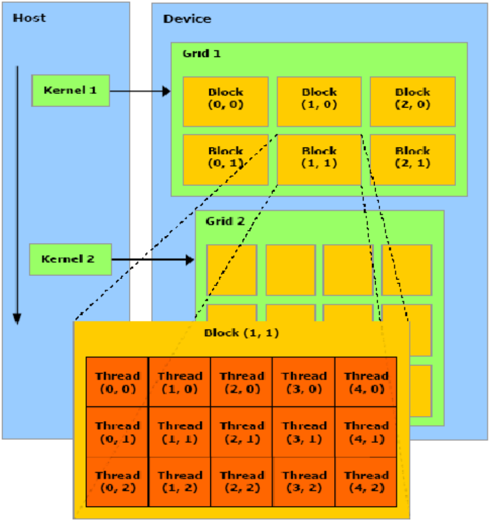
- Research opportunities exist in providing standard scientific computing package like BLAST on GPU computing.

Multi-GPU and OpenMP

- OpenMP
  - shared memory(fast cached)
  - high performance yet limited scalability
  - branching adaptability
- CUDA on GPU
  - many cores
  - shared memory (cached and un-cached)
  - fast on-board memory (register, local, share, global, const ,texture)
  - hardware thread management (zero overhead)
- 4 GPUs allowed on one computer. Affordable personal supercomputer.

CUDA Program Model



The host issues a succession of kernel invocations to the device. Each kernel is executed as a batch of threads organized as a grid of thread blocks

16 successive threads is called half-wrap, processed simultaneously on one SP.

- no diversity
- access successive memory in one command

Memory Model